OCR
Oxford Cambridge and RSA

Thursday 13 June 2019 – Afternoon

AS Level Further Mathematics B (MEI)

Y415/01 Mechanics b

Time allowed: 1 hour 15 minutes

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You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 A small object of mass 5 kg is attached to one end of each of two identical parallel light elastic strings. The upper ends of both strings are attached to a horizontal ceiling.
The object hangs in equilibrium at R, with the extension of each string being 0.1 m, as shown in Fig. 1.

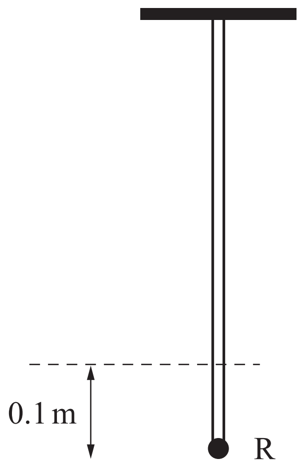

Fig. 1

(a) Find the stiffness of each string. **[3]**

One of the strings is now removed and the object initially falls downwards. The object does not return to R at any point in the subsequent motion.

(b) Suggest a reason why the object does not return to R. **[1]**

2 A particle P of mass m travels in a straight line on a smooth horizontal surface.
At time t, P is a distance x from a fixed point O and is moving with speed v away from O. A horizontal force of magnitude $3mt$ acts on P, in a direction away from O.

(a) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 3t$. **[1]**

(b) Verify that the general solution of this differential equation is $x = \frac{1}{2}t^3 + At + k$, where A and k are constants. **[2]**

(c) Given that $x = 6$ and $v = 12$ when $t = 1$, find the values of A and k. **[4]**

3 A particle Q of mass m moves in a horizontal plane under the action of a single force $\mathbf{F}$.

At time t, Q has velocity $\begin{pmatrix} 2 \\ 3t-2 \end{pmatrix}$.

(a) Find an expression for $\mathbf{F}$ in terms of m. **[2]**

At time t, the displacement of Q is given by $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$. When $t = 1$, Q is at the point with position vector $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$.

(b) Find the equation of the path of Q, giving your answer in the form $y = ax^2 + bx + c$, where a, b and c are constants to be determined. **[7]**

(c) What can you deduce about the path of Q from the value of the constant c you found in part **(b)**? **[1]**

4 Two uniform discs, A of mass 0.2 kg and B of mass 0.5 kg, collide with smooth contact while moving on a smooth horizontal surface.
Immediately before the collision, A is moving with speed $0.5\,\text{m s}^{-1}$ at an angle α with the line of centres, where $\sin\alpha = 0.6$, and B is moving with speed $0.3\,\text{m s}^{-1}$ at right angles to the line of centres. A straight smooth vertical wall is situated to the right of B, perpendicular to the line of centres, as shown in Fig. 4. The coefficient of restitution between A and B is 0.75.

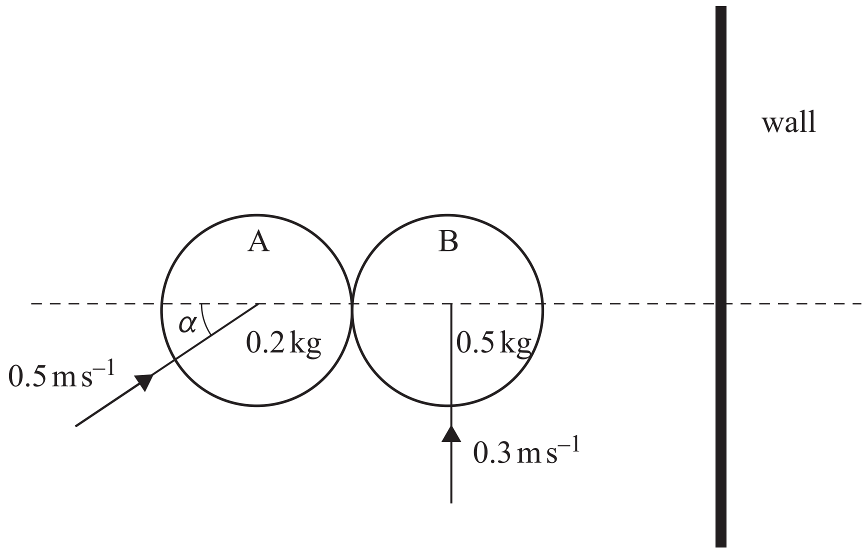

Fig. 4

(a) Find the speeds of A and B immediately after the collision. **[8]**

(b) Explain why there could be a second collision between A and B if B rebounds from the wall with sufficient speed. **[1]**

(c) Find the range of values of the coefficient of restitution between B and the wall for which there will be a second collision between A and B. **[3]**

(d) How does your answer to part **(b)** change if the contact between B and the wall is not smooth? **[1]**

5 Fig. 5 shows the curve with equation $y = -x^2 + 4x + 2$.
The curve intersects the x-axis at P and Q. The region bounded by the curve, the x-axis, the y-axis and the line $x = 4$ is occupied by a uniform lamina L. The horizontal base of L is OA, where A is the point (4, 0).

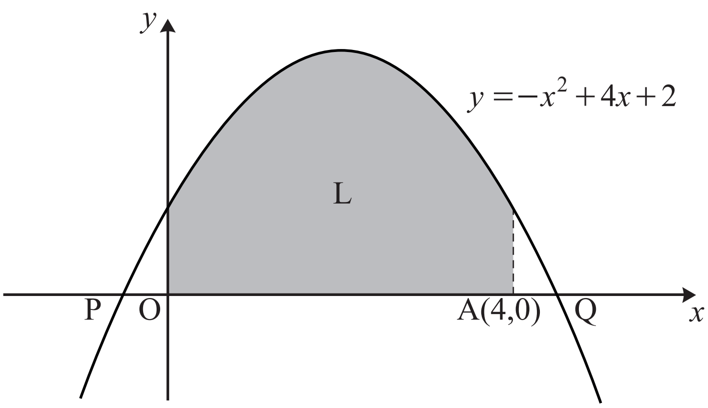

Fig. 5

(a) **(i)** Explain why the centre of mass of L lies on the line $x = 2$. **[1]**

(ii) **In this question you must show detailed reasoning.**

Find the y-coordinate of the centre of mass of L. **[7]**

(b) L is freely suspended from A. Find the angle AO makes with the vertical. **[2]**

The region bounded by the curve and the x-axis is now occupied by a uniform lamina M. The horizontal base of M is PQ.

(c) Explain how the position of the centre of mass of M differs from the position of the centre of mass of L. **[2]**

6 A smooth solid hemisphere of radius a is fixed with its plane face in contact with a horizontal surface.
The highest point on the hemisphere is H, and the centre of its base is O. A particle of mass m is held at a point S on the surface of the hemisphere such that angle HOS is 30°, as shown in Fig. 6.
The particle is projected from S with speed $0.8\sqrt{ag}$ along the surface of the hemisphere towards H.

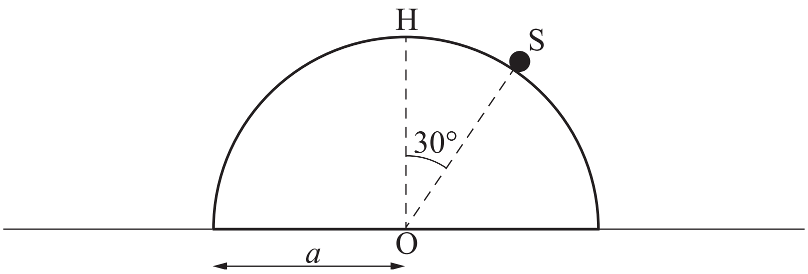

Fig. 6

(a) Show that the particle passes through H without leaving the surface of the hemisphere. **[4]**

After passing through H, the particle passes through a point Q on the surface of the hemisphere, where angle HOQ = $\theta°$.

(b) State, in terms of g and θ, the tangential component of the acceleration of the particle when it is at Q. **[1]**

The particle loses contact with the hemisphere at Q and subsequently lands on the horizontal surface at a point L.

(c) Find the value of $\cos\theta$ correct to 3 significant figures. **[4]**

(d) Show that OL = ka, where k is to be found correct to 3 significant figures. **[5]**

END OF QUESTION PAPER

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