



Oxford Cambridge and RSA

**Monday 24 June 2019 – Morning**

**A Level Further Mathematics B (MEI)**

**Y435/01 Extra Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The matrix  $\mathbf{A}$  is  $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$ .

(a) Given that  $\mathbf{A}$  represents a reflection, write down the eigenvalues of  $\mathbf{A}$ . [1]

(b) Hence find the eigenvectors of  $\mathbf{A}$ . [3]

(c) Write down the equation of the mirror line of the reflection represented by  $\mathbf{A}$ . [1]

2 A surface  $S$  is defined by  $z = 4x^2 + 4y^2 - 4x + 8y + 11$ .

(a) Show that the point  $P(0.5, -1, 6)$  is the only stationary point on  $S$ . [2]

(b) (i) On the axes in the Printed Answer Booklet, draw a sketch of the contour of the surface corresponding to  $z = 42$ . [2]

(ii) By using the sketch in part (b)(i), deduce that  $P$  must be a minimum point on  $S$ . [3]

(c) In the section of  $S$  corresponding to  $y = c$ , the minimum value of  $z$  occurs at the point where  $x = a$  and  $z = 22$ .

Find all possible values of  $a$  and  $c$ . [4]

3 The matrix  $\mathbf{A}$  is  $\begin{pmatrix} -1 & 2 & 4 \\ 0 & -1 & -25 \\ -3 & 5 & -1 \end{pmatrix}$ .

Use the Cayley-Hamilton theorem to find  $\mathbf{A}^{-1}$ . [8]

4  $T$  is the set  $\{1, 2, 3, 4\}$ . A binary operation  $\bullet$  is defined on  $T$  such that  $a \bullet a = 2$  for all  $a \in T$ . It is given that  $(T, \bullet)$  is a group.

(a) Deduce the identity element in  $T$ , giving a reason for your answer. [2]

(b) Find the value of  $1 \bullet 3$ , showing how the result is obtained. [3]

(c) (i) Complete a group table for  $(T, \bullet)$ . [2]

(ii) State with a reason whether the group is abelian. [1]

- 5 A financial institution models the repayment of a loan to a client in the following way.
- An amount,  $\pounds C$ , is loaned to the client at the start of the repayment period.
  - The amount owed  $n$  years after the start of the repayment period is  $\pounds L_n$ , so that  $L_0 = C$ .
  - At the end of each year, interest of  $\alpha\%$  ( $\alpha > 0$ ) of the amount owed at the start of that year is added to the amount owed.
  - Immediately after interest has been added to the amount owed a repayment of  $\pounds R$  is made by the client.
  - Once  $L_n$  becomes negative the repayment is finished and the overpayment is refunded to the client.
- (a) Show that during the repayment period,  $L_{n+1} = aL_n + b$ , giving  $a$  and  $b$  in terms of  $\alpha$  and  $R$ . [2]
- (b) Find the solution of the recurrence relation  $L_{n+1} = aL_n + b$  with  $L_0 = C$ , giving your solution in terms of  $a$ ,  $b$ ,  $C$  and  $n$ . [5]
- (c) Deduce from parts (a) and (b) that, for the repayment scheme to terminate,  $R > \frac{\alpha C}{100}$ . [2]

A client takes out a  $\pounds 30\,000$  loan at 8% interest and agrees to repay  $\pounds 3000$  at the end of each year.

- (d) (i) Use an algebraic method to find the number of years it will take for the loan to be repaid. [3]
- (ii) Taking into account the refund of overpayment, find the total amount that the client repays over the lifetime of the loan. [3]
- 6 (a) Given that  $\sqrt{7}$  is an irrational number, prove that  $a^2 - 7b^2 \neq 0$  for all  $a, b \in \mathbb{Q}$  where  $a$  and  $b$  are not both 0. [2]
- (b) A set  $G$  is defined by  $G = \{a + b\sqrt{7} : a, b \in \mathbb{Q}, a \text{ and } b \text{ not both } 0\}$ .  
Prove that  $G$  is a group under multiplication. (You may assume that multiplication is associative.) [7]
- (c) A subset  $H$  of  $G$  is defined by  $H = \{1 + c\sqrt{7} : c \in \mathbb{Q}\}$ .  
Determine whether or not  $H$  is a subgroup of  $(G, \times)$ . [2]
- (d) Using  $(G, \times)$ , prove by counter-example that the statement ‘An infinite group cannot have a non-trivial subgroup of finite order’ is false. [2]

**END OF QUESTION PAPER**

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