OCR
Oxford Cambridge and RSA

Tuesday 19 October 2021 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Time allowed: 2 hours 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Section A (34 marks)

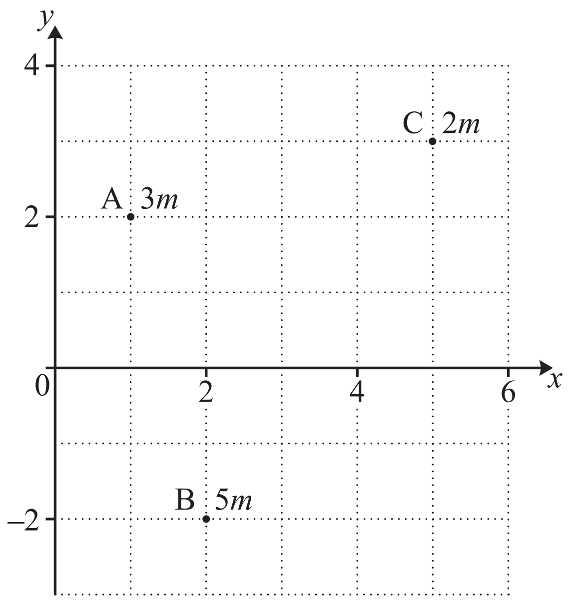
Answer **all** the questions.

1 A small ball of mass 0.25 kg is held above a horizontal floor. The ball is released from rest and hits the floor with a speed of $5\,\mathrm{m\,s^{-1}}$. It rebounds from the floor with a speed of $4.2\,\mathrm{m\,s^{-1}}$.

The situation is modelled by assuming that the ball is in contact with the floor for 0.02 s and during this time the normal contact force the floor exerts on the ball is constant.

Determine the magnitude of the normal contact force that the floor exerts on the ball. **[3]**

2 The diagram shows a system of three particles of masses $3m$, $5m$ and $2m$ situated in the x-y plane at the points A(1, 2), B(2, −2) and C(5, 3) respectively.



Determine the coordinates of the centre of mass of the system of particles. **[4]**

3 One end of a light elastic spring of natural length 0.3 m is attached to a fixed point. A mass of 4 kg is attached to the other end of the spring.

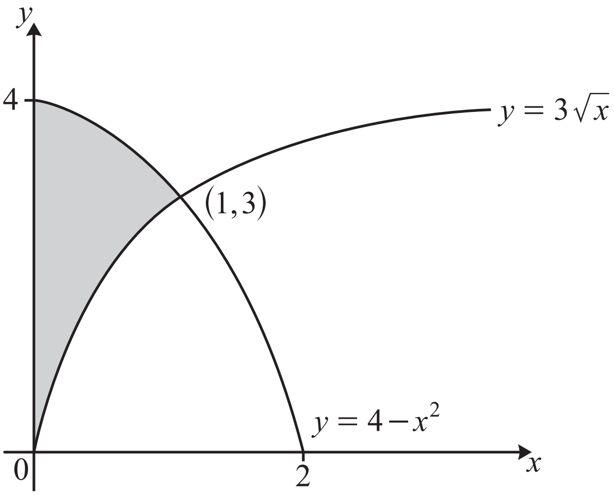
When the spring hangs vertically in equilibrium the extension of the spring is 0.02 m.

(a) Determine the modulus of elasticity of the spring. **[3]**

A student calculates that if the mass of 4 kg is removed and replaced with a mass of 20 kg the extension of the spring will be 0.1 m.

(b) Suggest a reason why this extension may not be 0.1 m. **[1]**

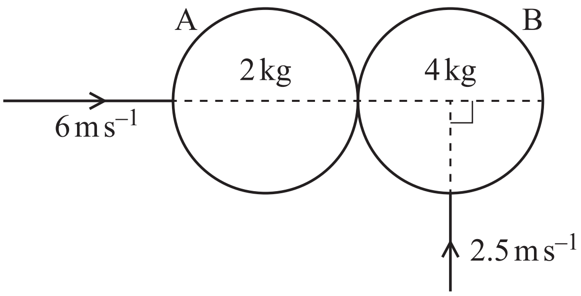
4 In this question you must show detailed reasoning.



The diagram shows parts of the curves $y = 3\sqrt{x}$ and $y = 4 - x^2$, which intersect at the point (1, 3). The shaded region, bounded by the two curves and the y-axis, is occupied by a uniform lamina.

Determine the exact x-coordinate of the centre of mass of the lamina. **[6]**

5 Two small uniform smooth spheres A and B, of equal radius, have masses 2 kg and 4 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed $6\,\mathrm{m\,s^{-1}}$ and is moving along the line of centres, and B has speed $2.5\,\mathrm{m\,s^{-1}}$ and is moving along a line which is perpendicular to the line of centres (see diagram).



The direction of motion of B after the collision makes an angle of 45° with the line of centres.

Determine the coefficient of restitution between A and B. **[6]**

6 **(a)** Write down the dimensions of force. **[1]**

The force F of gravitational attraction between two objects with masses m_1 and m_2, at a distance d apart, is given by

$$F = \frac{Gm_1m_2}{d^2},$$

where G is the universal gravitational constant.

In SI units the value of G is $6.67 \times 10^{-11} \mathrm{kg}^{-1}\mathrm{m}^3\mathrm{s}^{-2}$.

(b) Write down the dimensions of G. **[1]**

(c) Determine the value of G in imperial units based on pounds, feet, and seconds.

Use the facts that 1 pound = 0.454 kg and 1 foot = 0.305 m. **[2]**

For a planet of mass M and radius r, it is suggested that the velocity v needed for an object to escape the gravitational pull of the planet, the 'escape velocity', is given by the following formula.

$$v = \sqrt{\frac{kGM}{r}},$$

where k is a dimensionless constant.

(d) Show that this formula is dimensionally consistent. **[3]**

Information regarding the planets Earth and Mars can be found in the table below.

	Earth	Mars
Radius (m)	6 371 000	3 389 500
Mass (kg)	5.97×10^{24}	6.39×10^{23}
Escape velocity ($\mathrm{m\,s^{-1}}$)	11 186	

(e) Using the formula $v = \sqrt{\frac{kGM}{r}}$, determine the escape velocity for planet Mars. **[4]**

Section B (86 marks)

7 A box B of mass m kg is raised vertically by an engine working at a constant rate of kmg W. Initially B is at rest. The speed of B when it has been raised a distance x m is denoted by $v\,\mathrm{m\,s^{-1}}$.

(a) Show that $v^2\dfrac{\mathrm{d}v}{\mathrm{d}x} = (k-v)g$. **[3]**

(b) Verify that $gx = k^2\ln\left(\dfrac{k}{k-v}\right) - kv - \dfrac{1}{2}v^2$. **[5]**

(c) By using the work-energy principle, show that the time taken for B to reach a speed $V\,\mathrm{m\,s^{-1}}$ from rest is given by

$$\frac{k}{g}\ln\left(\frac{k}{k-V}\right) - \frac{V}{g}.$$ **[4]**

8 A capsule consists of a uniform hollow right circular cylinder of radius r and length $2h$ attached to two uniform hollow hemispheres of radius r.
The centres of the plane faces of the hemispheres coincide with the centres, A and B, of the ends of the cylinder.

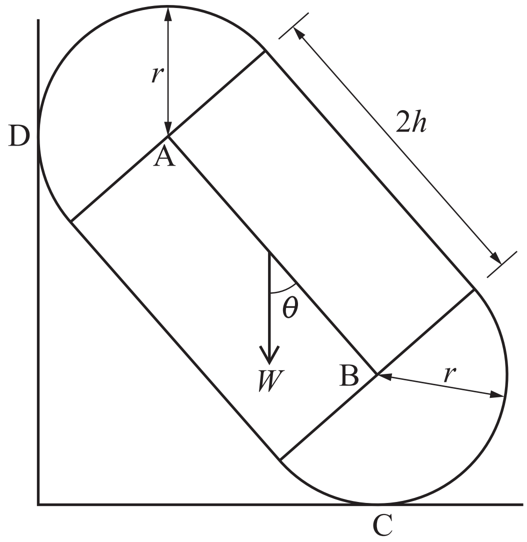


Fig. 8

Fig. 8 represents a vertical cross-section through a plane of symmetry of the capsule as it rests in limiting equilibrium with a point C of one hemisphere on a rough horizontal floor and a point D of the other hemisphere against a rough vertical wall.

The total weight of the capsule is W and acts at a point midway between A and B. The plane containing A, B, C and D is vertical, with AB making an acute angle θ with the downward vertical.

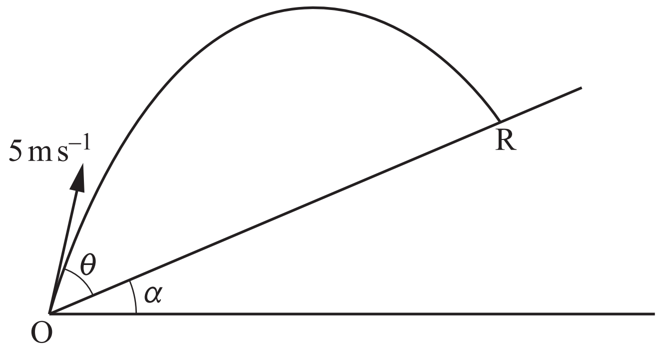
(a) Complete the copy of **Fig. 8** in the Printed Answer Booklet to show all the remaining forces acting on the capsule. **[1]**

The coefficient of friction at each point of contact is $\frac{1}{3}$.

(b) By resolving vertically and horizontally, determine the magnitude of the normal contact force between the floor and the capsule in terms of W. **[5]**

(c) By determining an expression for r in terms of h and θ, show that $\tan\theta > \frac{3}{4}$. **[6]**

9 A small ball P is projected with speed $5\,\mathrm{m\,s^{-1}}$ at an angle of elevation of $(\alpha + \theta)$ from a point O at the bottom of a plane inclined at α to the horizontal. P subsequently hits the plane at a point R, where OR is a line of greatest slope, as shown in the diagram.



(a) By deriving an expression, in terms of θ, α and g, for the time of flight of P, show that the distance OR, in metres, is

$$\frac{50\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}.$$ **[8]**

(b) By using the identity $2\sin A\cos B \equiv \sin(A+B) - \sin(B-A)$, determine, in terms of g and $\sin\alpha$, an expression for the maximum range of P up the plane, as θ varies. **[3]**

(c) Given that OR is the maximum range of P up the plane and is equal to 1.8 m, determine the value of θ.

[4]

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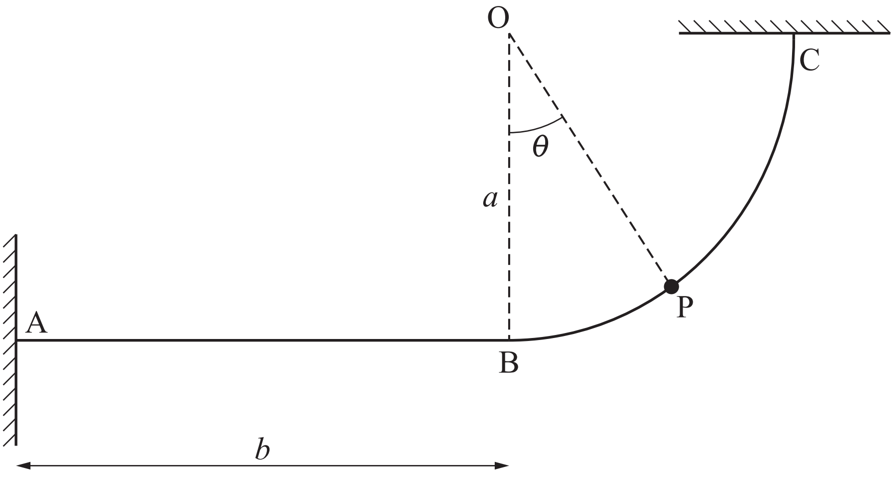

A rigid wire ABC is fixed in a vertical plane. The section AB of the wire, of length b, is straight and horizontal. The section BC of the wire is smooth and in the form of a circular arc of radius a and length $\frac{1}{2}a\pi$. The centre of the arc is O, which is vertically above B.

A bead P of mass m is threaded on the wire and projected from B with speed u towards C. The angle BOP when P is between B and C is denoted by θ, as shown in the diagram.

(a) Determine the magnitude of the normal reaction of the wire on P in terms of m, g, a, u and θ, when P is between B and C. **[7]**

P collides with a fixed barrier at C. The coefficient of restitution between P and the fixed barrier is e. After this collision P moves back towards B.

On the straight portion BA, the motion of P is resisted by a constant horizontal force F.

(b) Show that P will reach A if

$$Fb \leqslant \frac{1}{2}m\left[e^2u^2 + k(1-e^2)ga\right],$$

where k is an integer to be determined. **[6]**

11 Two small uniform smooth spheres A and B, of equal radius, have masses 4 kg and 3 kg respectively. The spheres are placed in a smooth horizontal circular groove. The coefficient of restitution between the spheres is e, where $e > \frac{2}{5}$.

At a given instant B is at rest and A is set moving along the groove with speed $V \mathrm{~m~s^{-1}}$. It may be assumed that in the subsequent motion the two spheres do not leave the groove.

(a) Determine, in terms of e and V, the speeds of A and B immediately after the first collision. **[6]**

(b) Show that the arc through which A moves between the first and second collisions subtends an angle at the centre of the circular groove of

$\frac{2\pi(4-3e)}{7e}$ radians. **[4]**

(c) **(i)** Determine, in terms of e and V, the speed of B immediately after the second collision. **[5]**

(ii) What can be said about the motion of A and B if the collisions between A and B are perfectly elastic? **[1]**

12 A particle P of mass m is fixed to one end of a light elastic string of natural length l and modulus of elasticity $12mg$. The other end of the string is attached to a fixed point O. Particle P is held next to O and then released from rest.

(a) Show that P next comes instantaneously to rest when the length of the string is $\frac{3}{2}l$. **[5]**

The string first becomes taut at time $t = 0$. At time $t \geqslant 0$, the length of the string is $l + x$, where x is the extension in the string.

(b) Show that when the string is taut, x satisfies the differential equation

$\ddot{x} + \omega^2 x = g$, where $\omega^2 = \frac{12g}{l}$. **[3]**

(c) By using the substitution $x = y + \frac{g}{\omega^2}$, solve the differential equation to show that the time when the string first becomes slack satisfies the equation

$\cos\omega t - \sqrt{k}\sin\omega t = 1$,

where k is an integer to be determined. **[10]**

END OF QUESTION PAPER

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