



Pearson  
Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics  
AS Further Decision 1 Paper 8FM0\_27

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS



### General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.  
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
  6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)		B1	1.2
		(1)	
(b)(i)	A semi-Eulerian graph contains <u>exactly two nodes of odd order</u> (and any number of nodes of even order)	B1	2.5
(b)(ii)	e.g. (two semi-Eulerian subgraphs of $K_5$ with a different number of edges) 	B1 B1	1.1b 1.1b
		(3)	
(c)	e.g. The graph with five vertices has $\frac{1+2+2+3+4}{2} = 6$ arcs but a tree on five nodes would contain only 4 arcs	B1 B1dep	2.2a 2.4
		(2)	
<b>(6 marks)</b>			
<b>Notes</b>			
<p>(a) <b>B1:</b> CAO (give bod for position of nodes)</p> <p>(b)(i) <b>B1:</b> CAO (accept ‘there are exactly two odd nodes’ but must contain exact oe (e.g. ‘only two odd nodes’ or ‘all but 2 nodes have an even order’ but not ‘the graph has two odd nodes’))</p> <p>(b)(ii) <b>B1:</b> One correct semi-Eulerian subgraph of <math>K_5</math> with five nodes <b>B1:</b> Two correct semi-Eulerian subgraphs of <math>K_5</math> with five nodes – note that the graphs must have a different number of edges</p> <p>(c) <b>B1:</b> Deducing that the graph has 6 arcs <b>or</b> a tree on five nodes has 4 arcs <b>or</b> the node of order 4 must be connected to the other 4 nodes <b>or</b> an argument based on the sum of the orders of both the graph and the tree (but must relate the orders to the number of arcs and not the number of nodes) <b>or</b> the node with order 4 and one of the nodes of orders 2 or 3 would create a cycle <b>or</b> a tree must have two nodes of order 1 <b>B1dep:</b> Complete argument – graph has 6 arcs and the tree would only have 4 arcs <b>or</b> the sum of the orders is 12 compared to 8 for the tree <b>or</b> the node of order 4 must be connected to the other 4 nodes therefore all the other vertices would have to have order 1 <b>or</b> the graph has 6 arcs and therefore with 5 vertices there would have to be cycles <b>or</b> the node of order 4 is connected to the other 4 nodes and so together with the node of order 3 (or 2) a cycle would be formed <b>or</b> a tree must have at least two nodes of order 1 as otherwise a cycle would be formed Note: no marks in (c) for attempts based only on examples of graphs drawn with the vertex orders as stated</p>			

Question	Scheme	Marks	AOs																																																								
2(a)	(i)																																																										
	<table border="1"> <thead> <tr> <th>A</th> <th>B</th> <th>N</th> <th>H</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> <td>4</td> <td>0.5</td> <td>0.25</td> <td>0</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>82</td> <td>1</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>1.5</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>92.125</td> <td>2</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>124.125</td> <td>2.5</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>202.25</td> <td>3</td> <td>50.5625</td> </tr> </tbody> </table>	A	B	N	H	C	D	E	F	1	3	4	0.5	0.25	0								82	1								1.5							92.125	2							124.125	2.5							202.25	3	50.5625	M1	1.1b
	A	B	N	H	C	D	E	F																																																			
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(ii) Final output = 50.5625	A1	1.1b																																																									
		(4)																																																									
(b)	$\int_1^3 x^4 dx = 48.4$	B1	1.1b																																																								
	$\left(\frac{50.5625 - 48.4}{48.4}\right) \times 100$	M1	1.1a																																																								
	4.47%	A1ft	3.2b																																																								
		(3)																																																									
<b>(7 marks)</b>																																																											
<b>Notes</b>																																																											
<p><b>(a)(i)</b></p> <p><b>M1:</b> At least three rows of cells completed (so at least two values of D and E given) with either a correct first row or 82 found for D – condone repeated values in all columns or a single value in each row</p> <p><b>A1:</b> CAO – the values in the second, third and fourth rows correct (so up to the 92.125 in column D and the 2 in column E) – accept exact equivalent fractions</p> <p><b>A1:</b> CAO – all values correct in columns A to E – accept exact equivalent fractions</p> <p><b>(ii)</b></p> <p><b>A1:</b> CAO (output = 50.5625) (or equivalent e.g. <math>50\frac{9}{16}</math>) – allow if stated only in column F</p> <p><b>(b)</b></p> <p><b>B1:</b> CAO (48.4)</p> <p><b>M1:</b> Correct method (including multiplying by 100) using candidate’s final output from (a)(ii) and their value for <math>I</math></p> <p><b>A1ft:</b> Follow through their final output from (a)(ii) (for reference: 4.4679752...) must be using 48.4 - dependent on M mark in (a) and percentage error being &lt; 10% (answer must be given to 3 significant figures)</p>																																																											

Question	Scheme	Marks	AOs
3(a)		M1 A1 A1 A1 A1 <b>(5)</b>	1.1b 1.1b 1.1b 1.1b 1.1b 
(b)	Activity F (and/or G) requires activity B and the two activities A and C to be completed before F (and/or G) can begin. The time to complete A and C is double that of B and so B can be delayed waiting for A and C to be completed and so B is therefore not critical.	B1 <b>(1)</b>	2.4 
(c)	Activities D, E and H	B1 <b>(1)</b>	2.2a 
<b>(7 marks)</b>			



## Notes

In (a) condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'G dealt with correctly' requires the correct precedences for this activity, i.e. B and C labelled correctly and leading into the same node and G starting from that node but do not consider the end node for G.

### Activity on node is M0

If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

### (a)

**M1:** At least eight activities (labelled on arc), one start, and at least two dummies placed

**A1:** Activities A – G dealt with correctly (bod if no arrow on activity C)

**A1:** First two required dummies + arrows dealt with correctly

**A1:** Activities H – K dealt with correctly (A0 if no arrows on preceding dummies (oe))

**A1:** CSO – Final required dummy + all arrows present and correctly placed with one finish and no additional dummies. Note that the arrow for the final dummy could be reversed. Note that there are several correct viable positions for the final dummy

**Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned**

### (b)

**B1:** CAO - some mention of the time required to complete A + C compared with B (for the next activity to begin (either F and/or G)) oe e.g. paths through B have a maximum length of 3 (non-dummy) activities and there is at least one path of length 4 which does not include B so B cannot be critical **OR** the late time for B must be the same as the late time for A + C which is twice the duration of B and therefore B is not critical. Give bod to responses that imply that B and C meet at the same event, but C is also dependent on A (the key point for awarding this mark is that activities A and C imply that B is not critical)

### (c)

**B1:** All three correct with no extras (ignore any mention of activity B)

Question	Scheme	Marks	AOs
4(a)	<p>Attempt to form a pair of simultaneous equations using their three working values from H  e.g. <math>3x + y = 15</math> or <math>x + y = 9</math>  <math>x = 3, y = 6</math></p>	M1	1.1b
		A1	1.1b
		A1ft	1.1b
		A1	1.1b
		M1dep A1 A1	3.1a 2.1 2.2a
		(7)	
(b)	Arcs BC and CD need to be traversed twice	B1	1.1b
		(1)	
(c)	Vertex C would appear 4 times	B1	2.2a
		(1)	
(d)	$135 + 4(3) + 2(6) + 12 = 171$	B1ft	1.1b
		(1)	
<b>(10 marks)</b>			

## Notes

(a)

**M1:** For a larger number replaced by a smaller one in the working value boxes at C, F or G

**A1:** For all values correct (and in correct order) at A, D, B and C (condone order of labelling starting at A with 0)

**A1ft:** For all values correct (and in correct order) at G and F following through from A, D, B and C

**A1:** For all **working values** correct at E and H (order of working values must be correct at E but condone any order of working values at H) **however, at H if only one working value is seen e.g.  $18 + 3x + y$  then both  $33$  and  $24 + x + y$  must be seen (or clearly implied) in later working for this mark to be awarded (e.g.  $3x + y = 15$  and  $x + y = 9$  would imply this). Similarly, if only two working values seen (e.g.  $18 + 3x + y$  and  $33$ ) at H then the third ( $24 + x + y$ ) must be implied by later working. Any incorrect working values seen at H though will score A0**

**M1dep:** Forming two equations from the candidate's three working values at H

(so two of their  $18 + 3x + y = 24 + x + y$ ,  $18 + 3x + y = 33$  and  $24 + x + y = 33$ ) – allow all three working values stated anywhere in their solution – dependent on previous M mark. Must be a complete method – so for those finding  $x$  from  $18 + 3x + y = 24 + x + y$  they must also either state or use one of the other two equations (so candidates must be interacting with all three paths from A to H)

**A1:** Two correct equations formed (dependent on correct working values either seen at H or in their subsequent working) – can be unsimplified but must come from correct working

**A1:** CAO for  $x$  and  $y$  ( $x = 3$  and  $y = 6$ ) – must come from correct working

If all three correct working values at H are seen (either at H or subsequent working) together with both correct answers (with no other working) then award M1A1A1.

(b)

**B1:** CAO (arcs BC and CD)

(c)

**B1:** CAO (4 times)

(d)

**B1ft:** Follow through only for  $135 + 12 + 4x + 2y$  (for their  $x$  and  $y$  values provided  $7 < x + y < 20$  and  $x$  and  $y$  are positive constants)

Question	Scheme	Marks	AOs
<b>5(a)</b>	Minimise ( $P =$ ) $x + 5y + 4z$	B1	3.3
	Subject to $x \geq \frac{3}{5}(x + y + z) (\Rightarrow 2x \geq 3y + 3z)$	B1	3.3
	$3y \geq 2z$	B1	3.3
	$x + y + z = 1000$	B1	3.3
	$z = 1000 - x - y$ substituted into objective and constraints gives	M1	3.1a
	Minimise ( $P =$ ) $y - 3x (+ 4000)$ subject to $x \geq 600$ and $2x + 5y \geq 2000$	A1 A1	1.1b 1.1b
		<b>(7)</b>	
<b>(b)</b>	(i) Using least value of $x$ to find $y$ and $z$ 600 roses, 160 hydrangeas and 240 peonies	M1 A1	3.4 3.2a
	(ii) £2360	A1	1.1b
		<b>(3)</b>	
<b>(10 marks)</b>			
<b>Notes</b>			
<p><b>(a)</b></p> <p><b>B1:</b> CAO (for objective) – must contain ‘minimise’ or ‘min’ only (so not ‘minimum’) either when stated in terms of <math>x</math>, <math>y</math> and <math>z</math> or <math>x</math> and <math>y</math> only</p> <p><b>B1:</b> <math>x \geq \frac{3}{5}(x + y + z)</math> oe – need not be simplified for this mark, accept <math>x \geq \frac{3}{5}(1000)</math></p> <p><b>B1:</b> <math>3y \geq 2z</math> or any equivalent form (need not be simplified nor integer coefficients for this mark)</p> <p><b>B1:</b> <math>x + y + z = 1000</math> (could be implied by earlier/late working)</p> <p><b>M1:</b> Eliminating <math>z</math> from either the objective <b>or</b> both constraints using the constraint <math>x + y + z = 1000</math></p> <p><b>A1:</b> Correct objective in terms of <math>x</math> and <math>y</math> only – condone lack of ‘minimise’</p> <p><b>A1:</b> Both constraints correct (<math>x \geq 600</math> and <math>2x + 5y \geq 2000</math> - must be integer coefficients for this mark)</p>			
<p><b>(b)(i)</b></p> <p><b>M1:</b> Using their least value of <math>x</math> to find both <math>y</math> and <math>z</math> (with both <math>y</math> and <math>z</math> being positive integers) – note that all values must satisfy the constraint <math>x + y + z = 1000</math> (and must all be integers)</p> <p><b>A1:</b> All three types of flowers correct (in context – so not just in terms of <math>x</math>, <math>y</math> and <math>z</math>) – must come from correct constraints in (a)</p> <p><b>(ii)</b></p> <p><b>A1:</b> CAO for cost (condone lack of units but not 2360p) – must come from correct constraints in (a)</p> <p><b>SC for (b)</b> – for those candidates with the constraint <math>2y \geq 3z</math> in (a) leading to 600 roses, 240 hydrangeas and 160 peonies (so not just in terms of <math>x</math>, <math>y</math> and <math>z</math>) together with (£)2440 award <b>SC M1A1A0</b> in (b)</p>			

