

Molecular kinetic theory model

Derivation of kinetic theory equation

- Initial momentum of a gas particle is mv
Momentum after contact with the container wall is $-mv$
 $\Delta\text{momentum} = 2mv$

- Time between collisions: $v = \frac{s}{t} = \frac{2l_1}{t}$
Time between collisions, $t = \frac{2l_1}{v}$

- Newton's second law:

$$F = \frac{\Delta mv}{\Delta t} = 2mv \times \frac{v}{2l_1} = \frac{mv^2}{l_1}$$

- Pressure:

$$p = \frac{F}{A} = \frac{mv^2}{l_1} \times \frac{1}{l_2 l_3} = \frac{mv^2}{l_1 l_2 l_3} = \frac{mv^2}{V}$$

- However, there are **many** gas particles:

$$p = \frac{mv_1^2}{V} + \frac{mv_2^2}{V} + \frac{mv_3^2}{V} \dots + \frac{mv_n^2}{V}$$

$$p = \frac{m}{V}(v_1^2 + v_2^2 + v_3^2 \dots + v_n^2)$$

Need to use an average:

$$\text{average } v, (c_{RMS})^2 = \frac{v_1^2 + v_2^2 + v_3^2 \dots v_n^2}{n}$$

$$n(c_{RMS})^2 = v_1^2 + v_2^2 + v_3^2 \dots v_n^2$$

$$\therefore \text{overall: } p = \frac{mn(c_{RMS})^2}{V}$$

- However, particles do not just move in one plane \therefore we are dealing with a component in one of three planes - a third of the total magnitude.

$$p = \frac{mn(c_{RMS})^2}{3V}$$

\therefore

- $mn = \text{total mass, } M$:

$$p = \frac{M(c_{RMS})^2}{3V}$$

$$p = \frac{M}{V}$$

$$\therefore p = \frac{1}{3}p(c_{RMS})^2$$

Molecules and kinetic energy

For an ideal gas its internal energy is due only to the kinetic energy of the molecules of the gas.

$$\text{Kinetic energy of a molecule} = \frac{\text{total } E_k \text{ of all the molecules}}{\text{total number of molecules}} = \frac{\frac{1}{2}m(c_1^2 + c_2^2 \dots c_N^2)}{N} = \frac{1}{2}m(c_{RMS})^2$$

The higher the temperature of a gas the greater the mean kinetic energy of a molecule of the gas.

$$\text{for an ideal gas, mean } E_k \text{ of a molecule} = \frac{3}{2}kT$$

$$\text{total } E_k \text{ of } n \text{ mol of ideal gas} = \frac{3}{2}nRT = (\text{internal energy})$$