revisely

Gravitational fields

Gravitational fields

A force field is a region in which a body experiences a non-contact force. A force field can be represented as a vector, the direction of which must be determined by inspection.

- Gravity is a universal attractive force which acts between all matter
  - magnitude of a force between point masses, $F = \frac{Gm_1m_2}{r^2}$ where G is the gravitational constant

- A gravitational field can be represented by field lines - also known as lines of force.
  - This is the path followed by a small mass placed close to a massive body.
  - Note that for a radial field, the field lines point towards the centre. In a uniform field e.g. close to the Earth's surface, field lines act straight down - parallel to each other and evenly spaced.

- The gravitational field strength, g, is the force per unit mass on a small test mass placed in the field.
  $g = \frac{F}{m}$

- In a radial field, the magnitude of:
  $g = \frac{GM}{r^2}$

Gravitational potential

- **Gravitational potential** at a point is the gravitational potential energy per unit mass of a small test mass.
  - This is equal to the work done per unit mass to move an object from infinity (where potential = 0) to that point.

gravitational potential, $V = \frac{W}{m}$ unit: J kg^{-1}

work done moving mass m: $\Delta W = m\Delta V$

gravitational potential in a radial field: $V = -\frac{GM}{r}$

  - The negative sign is due to the reference point being infinity, and the fact that other than at infinity the force is in fact attractive.

- ΔV can be found from the area of a g-r graph
- **Equipotentials** are surfaces of constant potential - no work needs to be done to move along an equipotential surface.
- **Potential gradient** at a point in a gravitational field is the change of potential per metre at that point
- In general, for ΔV over a small distance Δr, potential gradient = $\frac{\Delta V}{\Delta r}$
- Gravitational field strength is the negative of potential gradient:

$$g = -\frac{\Delta V}{\Delta r}$$

Orbits and satellites

If an object is moving parallel to a planet's surface at the correct speed such that the centripetal force required is matched exactly by the force of gravity, it will orbit.

For a satellite orbiting at distance r from the centre of a planet: $\frac{GM_{planet}m}{r^2} = \frac{mv^2}{r} = mr\omega^2$ showing m irrelevant

- For **geostationary** orbit, $T_{sat} = T_{planet}$, so for earth T≈ 86 400 s

$$-T = \frac{2\pi}{\omega}$$

Kepler's 3rd Law proof and derivation:

- For an object in orbit around mass M:
  1. $\frac{GM}{r^2} = r\omega^2$ so $\frac{GM}{r^3} = \omega^2$
  2. Combining with $T = \frac{2\pi}{\omega}$ gives $\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$, or $T^2 = \frac{4\pi^2}{GM}r^3$
  3. Everything is constant except T and r, meaning $T^2 \propto r^3$ - Kepler's 3rd Law
  4. To further prove K3L, if $T^2 = \frac{4\pi^2}{GM}r^3$, taking logarithms gives $log(T^2) = log(\frac{4\pi^2}{GM}r^3)$
  5. $log(T^2) = log(\frac{4\pi^2}{GM}) + log(r^3)$
  6. $2log(T^2) = 3log(r) + log(\frac{4\pi^2}{GM})$
  7. $log(T^2) = 1.5log(r) + 0.5log(\frac{4\pi^2}{GM})$
  8. $log(T^2) = 1.5log(r) + log(\sqrt{\frac{4\pi^2}{GM}})$
  9. Hence a graph of log T against log r has gradient 1.5 and positive y-intercept of $\frac{2\pi}{\sqrt{GM}}$

Escape velocity

For an object to go into orbit once launched rather than fall back to Earth, it must never run out of kinetic energy. So supplied $\frac{E_k}{m} \geq V$

tic Equating E_k and $V \cdot m$ allows us to work out that:

$$\text{escape velocity, } v = \sqrt{\frac{2GM}{r}}$$

Energy considerations

A satellite $E_k = \frac{1}{2}mv^2$. Equating forces in orbit gives $\frac{mv^2}{r} = \frac{GMm}{r^2}$ or $v^2 = \frac{GM}{r}$

Hence to be in orbit, $E_k = \frac{GMm}{2r}$

Potential energy is calculated from gravitational potential: $E_p = -\frac{GM}{r} \cdot m$

The total energy is the sum: $E_T = \frac{GMm}{2r} + (-\frac{GMm}{r})$

$$E_T = -\frac{GMm}{2r}$$